Varianta 058

SUBIECTUL **I**

a) $\sqrt{5}$.

b) 1.

c) $\frac{\sqrt{2}}{2}$.

d) $B_1(0,1) \in M$, $A_2(2,0) \in M$.

e) $P\left(\frac{2}{3}, \frac{2}{3}\right)$.

f) 11.

SUBIECTUL **II**

1.

a) $x = 3$.

b) $x_1 + x_2 = 4$.

c) 2.

d) 10.

e) 15.

2.a) $\frac{-2x}{\left(x^2+1\right)^2}$.

b) $y = 1$ asimptotă orizontală spre $+\infty$.

c) $A(0,2)$ punct de maxim local.

d) $1+\frac{\pi}{4}$.

e) Cum $x_0 = 0$ este punct de maxim local rezultă că $f(x) \le f(0), \forall x \in$ **R.** Obținem $f(x) \le 2$, $\forall x \in$ **R.**

SUBIECTUL **III**

a) det A=$\begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix}$=1;det B=$\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$=1.Atunci det A=det B.

b) $A^2 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$. $B^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$.Deci $A^2 \ne B^2$.

c) $A \circ B = A \cdot B + B \cdot A$.

$A \cdot B = \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$; $B \cdot A = \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$.Deci $A \circ B = \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 10 \\ 0 & 2 \end{pmatrix}$.

d) Prin adunare obținem $2x+2y+2z=10$ sau $x+y+z=5$ .Obținem z=3 .Atunci $y=0$ iar $x=2$ .Deci , $S=\{(2,0,3)\}$.

e) $A^n = \begin{pmatrix} 1 & 3n \\ 0 & 1 \end{pmatrix}$ $\forall\ n \ge 1$.

$$U = = \begin{pmatrix} 6 & 63 \\ 0 & 6 \end{pmatrix}.$$

$\det U = 36$

f) $X \circ C = X \cdot \frac{1}{2} I_2 + \frac{1}{2} I_2 X = X$.

$C \circ X = \frac{1}{2} I_2 X + X \cdot \frac{1}{2} I_2 = X$ .Deci $X \circ C = C \circ X$.

g) $S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$**,** $T = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

SUBIECTUL **IV**

a) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{x}{e^x}$ =0. Obținem $y = 0$ asimptotă orizontală spre $+\infty$.

b) $f'(x) = \frac{e^x(1-x)}{e^{2x}}$.

c) Din $f'(x) = 0$ avem $x = 1$

x	$-\infty$		1		$+\infty$
$f'(x)$		$+$	0	-	
$f(x)$	$\nearrow$		$\frac{1}{e}$	$\searrow$	

Obținem $A\left(1, \frac{1}{e}\right)$ punct de maxim local .

d) Din **c)** avem $x_0 = 1$ este punct de maxim global.Rezultă $f(x) \le \frac{1}{e}$.

e) $\int\limits_0^1 f(x)dx = \int\limits_0^1 \frac{x}{e^x} dx = \int\limits_0^1 x(e^{-x})dx = \int\limits_0^1 x(-e^{-x})' dx = -xe^{-x}\Big|_0^1 - \int\limits_0^1 (-e^{-x})dx = -\frac{2}{e} + 1$.

f) $\lim\limits_{n\to\infty}\left(f(1) + \frac{1}{2} \cdot f(2) + \frac{1}{3} \cdot f(3) + .. + \frac{1}{n} \cdot f(n) \right) = \lim\limits_{n\to\infty}\left(\frac{1}{e^1} + \frac{1}{e^2} + \frac{1}{e^3} + .. + \frac{1}{e^n} \right) = \frac{1}{e-1}$.

g) $f(x) \le \frac{1}{e}, \forall\ x \in \mathbf{R}$, adică $x \cdot e \le e^x, \forall\ x \in \mathbf{R}$.

Pentru $x = \pi$ avem $e \cdot \pi < e^\pi$.